1. [S.6.2: p. 486: #52] Height of a Tower—A 600 ft guy wire is attached to the top of a communications tower. If the wire makes an angle of $65^\circ$ with the ground, how tall is the communications tower?

Solution

1. **Assumptions:** (1) The ground is level and (2) the tower is "straight up," i.e. perpendicular to the ground.

2. **Figure:**

3. Which trig ratio will help me?

$$\sin \theta = \sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 65^\circ = \frac{h}{600}$$

4. Now "do the math."

$$h = 600 \sin 65^\circ \approx 543.784 \text{ ft}$$

Notes: (*) Calculator approximation—remember to put the calculator into which mode?

(**) I rounded my final answer to 3 significant digits, because I assumed that the given value of 600 ft for the guy wire was accurate to 3 significant digits.

Final Answer: The tower is approximately 544 ft tall.

2. [S.6.3: p. 495: #6] Find the reference angle for each of the following: (a) $\frac{4\pi}{3}$, (b) $\frac{33\pi}{4}$, (c) $-\frac{23\pi}{6}$

Solutions

(a) $\theta = \frac{4\pi}{3}$

(b) $\theta = \frac{33\pi}{4}$

So $\frac{33\pi}{4}$ is 4 times around the circle plus 1 extra $\pi/4$. 

(c) $\theta = \frac{23\pi}{6}$

So $\frac{23\pi}{6}$ is $\frac{\pi}{3}$
\[ (C) \quad -\frac{23 \pi}{6} \quad \text{Soln.} \ 1 \]

\[ \frac{1}{12} \]

So \(-\frac{23 \pi}{6}\) is \(1\) complete negative revolution and "almost" another neg. revolution.

\[ \frac{1}{12} \]

The "dots" are each \(\frac{\pi}{6}\) units.

\[ \text{Fig.} \]

\[ \text{Soln.} \]

One method of solution is to simply remember the formula for the area of a triangle, given two sides and the included angle:

\[ A = \frac{1}{2} ab \sin \Theta \quad (\star) \quad (\text{page } 495) \]

Another approach is to recall that \(\frac{h}{7} = \sin(72^\circ) \Rightarrow h = (7) \sin(72^\circ) \quad (\star\star)\)

and to recall that the formula for the area of any triangle is \(A = \frac{1}{2} bh\) (where "b" stands for "base," and "h" stands for "height.") In our problem we have \(A = \frac{1}{2} (9) h\) \(\quad (\star\star\star)\)

Putting together (\(\star\star\)) \& (\(\star\star\star\)) we see \(A = \frac{1}{2} (9)(7) \sin(72^\circ)\), which is exactly the same as what you get by using formula (\(\star\)).

\[ A = \frac{63}{2} \sin(72^\circ) \approx 29.958 \ 280 \ 26 \approx 30 \ \text{unit}^2 \quad \text{[Note: I'll tell you how to round off.]} \]

\[ \text{The area of the triangle is approximately } 30 \text{ units}^2. \]
# 4 [S.G.8; p.497; # 65] Throwing a Shot Put - The range $R$ and the height $H$ of a shot put thrown with an initial velocity of $V_0 \text{ ft/s}$ at an angle $\theta$ are given by

$$ R = \frac{V_0^2 \sin(2\theta)}{g} \quad H = \frac{V_0^2 \sin^2(\theta)}{2g} $$

On the earth $g = 32 \text{ ft/s}^2$ and on the moon $g = 5.2 \text{ ft/s}^2$.

Find the range and height of a shot put thrown under the given conditions:
(a) On the earth with $V_0 = 12 \text{ ft/s}$ and $\theta = \frac{\pi}{6}$.
(b) On the moon with $V_0 = 12 \text{ ft/s}$ and $\theta = \frac{\pi}{6}$.

**Solution**

[Diagram of a shot put range and height with labels $R$ and $H$.]

(a) On earth: $g = 32 \text{ ft/s}^2$, $V_0 = 12 \text{ ft/s}$, $\theta = \frac{\pi}{6}$

$$ R = \frac{V_0^2 \sin(2\theta)}{g} = \left(12^2 \sin\left(2 \times \frac{\pi}{6}\right) \right) / 32 $$

$$ = \frac{144}{32} \sin\left(\frac{\pi}{3}\right) $$

$$ = \frac{9}{2} \sqrt{3} = \frac{9\sqrt{3}}{4} \text{ ft. (exact)} $$

$$ H = \frac{V_0^2 \sin^2(\theta)}{2g} = \left(12^2 \sin^2\left(\frac{\pi}{6}\right) \right) / 64 $$

$$ = \frac{144}{64} \sin^2\left(\frac{\pi}{6}\right) = \frac{9}{4} \left(\frac{1}{2}\right)^2 = \frac{9}{16} \text{ ft. (exact)} $$

(b) On moon: $g = 5.2 \text{ ft/s}^2$, $V_0 = 12 \text{ ft/s}$, $\theta = \frac{\pi}{6}$

$$ R = \frac{V_0^2 \sin(2\theta)}{g} = \left(12^2 \sin\left(2 \times \frac{\pi}{6}\right) \right) / 5.2 $$

$$ = \frac{144}{5.2} \sqrt{3} = \frac{180}{13} \sqrt{3} \text{ ft. (exact)} $$

$$ H = \frac{V_0^2 \sin^2(\theta)}{2g} = \left(12^2 \sin^2\left(\frac{\pi}{6}\right) \right) / 10.4 $$

$$ = \frac{144}{10.4} \text{ ft. (exact)} $$

cont...
More calculations:

1. \( \frac{9\sqrt{3}}{4} \approx 3.89714317 \approx 3.9 \text{ ft} \).

2. \( \frac{9}{16} = 0.5625 \text{ ft} \).

3. \( \frac{180}{13}\sqrt{3} \approx 23.98224195 \approx 24 \text{ ft} \).

4. \( \frac{45}{13} \approx 3.461538462 \approx 3.5 \text{ ft} \).

On earth the shot put will travel \( \frac{9\sqrt{3}}{4} \text{ ft} \) downrange, or approximately 3.9 ft, and it will reach a height of \( \frac{9}{16} \text{ ft} \) or 0.5625 ft.

On the moon the shot put will travel \( \frac{180}{13}\sqrt{3} \text{ ft} \) downrange, or approximately 24 ft, and it will reach a height of \( \frac{45}{13} \text{ ft} \), or approximately 3.5 ft.

That's all for now.