A. Background - Can you integrate?

$$\int x^2 e^x \, dx$$

Folley's method

$$\frac{d}{dx} \left( \frac{x^2}{e^x} \right) + \int e^x \, dx = 2x e^x - 2x e^x + 2e^x + C$$

B. Solve: \( \frac{dy}{dx} = x^2 y e^x \) \((\ast)\)

Solution: By way of §2.2 - use sep. of var. \((\ast)\) becomes

$$\frac{dy}{y} = x^2 e^x \, dx$$

$$\int \frac{1}{y} \, dy = \int x^2 e^x \, dx$$

$$\ln |y| = x^2 e^x - 2x e^x + 2e^x + C$$

$$\ln |y| = (x^2 - 2x + 2)e^x + C$$

II. Justification for Sep. of Var. (p. 33).

If we can write our ODE as

$$\frac{dy}{dx} = f(x) g(y) \quad \text{or} \quad \frac{f(x)}{h(y)}$$

then

$$h(y) \frac{dy}{dx} = f(x)$$ \((\ast)\)
If \( \phi(x) \) is a soln to \((*)\), then
\[
h(\phi(x)) \frac{d\phi(x)}{dx} = f(x)
\]
\[
\int h(\phi(x)) \phi'(x) \, dx = \int f(x) \, dx
\]
Let \( u = \phi(x) \), \( du = \phi'(x) \, dx \)
\[
\int h(u) \, du = \int f(x) \, dx
\]
Thus instead of going through this every time,
we just observe that we get the same result
by taking \((*)\) \( h(y) \, dy = f(x) \, dx \) and "clearing the fraction, then integrating"
\[
h(y) \, dy = f(x) \, dx
\]
\[
\int h(y) \, dy = \int f(x) \, dx
\]

\[\text{Example: [§ 2.2: p. 38: #19]}\]
Solve.
\[y \ln x \frac{dx}{dy} = \left( \frac{y+1}{x} \right)^2\]
\[\text{Solv:}\]
1. Sep. Var.: \( x^2 \ln x \, dx = \frac{y^2 + 2y + 1}{y} \, dy \)
2. \( x^2 \ln x \, dx = (y+2+\frac{1}{y}) \, dy \)
3. Integrate - leave it to you!
§ 2.3: Homogeneous Equations (in the sense of Euler)

(A) The concept of a **Homogeneous Function**.

The function \( f(x, y) \) is said to be **Homogeneous** of degree \( n \) iff

\[
f(tx, ty) = t^n f(x, y)
\]

(B) Example: \( f(x, y) = x^2 + xy \), then testing for homogeneity:

\[
f(tx, ty) = (tx)^2 + (tx)(ty)
= t^2 x^2 + t^2 xy
= t^2(x^2 + xy) = t^2 f(x, y).
\]

\[
\therefore \text{This fun. is homogeneous of degree 2}
\]