**Another Mixture Problem — §3.2: p. 91: #24**

**Data:**
- Large Tank w/ 500 gal pure water.
- Brine w/ 2 lb salt/gal pumped in @ 5 gpm.
- Well-mixed solution pumped out @ 10 gpm.

**Questions:**
- Find 
  \[ A(t) \] — am\(\bar{t}\) A (lb) of salt in tank @
  time \(t\) (min)
- When is the tank empty?

**Solution:**

\[
5 \frac{\text{gal}}{\text{min}} \cdot 2 \frac{\text{lb salt}}{\text{gal}} = 10 \frac{\text{lb}}{\text{min}}
\]

Gal in tank @ time \(t\)

\[500 - 5t\]

\[
10 \frac{\text{gal}}{\text{min}} \cdot \frac{A \text{ lb salt}}{500 - 5t \text{ gal}} = \frac{A \text{ lb}}{50 - 0.5t} \text{ lb/min}
\]

**Model: (I know about rates)**

\[
\frac{dA}{dt} = R_{\text{In}} - R_{\text{Out}}
\]

\[
\frac{dA}{dt} = 10 - \frac{A}{50 - 0.5t} \text{ (lb/min)}
\]

**Solve the Model.**

\[A' + \frac{1}{50 - 0.5t} A = 10\]

**Solution**

(a) Find \(\int \frac{1}{50 - 0.5t} dt\)

\[u\text{-subs: } \text{Let } u = 50 - 0.5t \Rightarrow du = -0.5 dt \]

\[\Rightarrow -2 \, du = dt\]
\[ \int_{50-0.5t}^{1} dt = -2 \int \frac{du}{u} = -2 \ln |u| + C \]

\[ = -2 \ln |50-0.5t| \quad \text{Let } C = 0 \]

\[ = \ln (50-0.5t)^{-2} \]

\[ \Rightarrow \mu(t) = e^{\ln (50-0.5t)^{-2}} = (50-0.5t)^{-2} \]

\[ \text{Apply } \mu(t): \]

\[ (50-0.5t)^{-2} A' + (50-0.5t)^{-2} \cdot \frac{1}{50-0.5t} A \]

\[ = (50-0.5t)^{-2} \cdot 10 \]

\[ (50-0.5t)^{-2} A' + (50-0.5t)^{-3} A = 10 (50-0.5t)^{-2} \]

\[ \left[ (50-0.5t)^{-2} A \right]' = 10 (50-0.5t)^{-2} \]

\[ (50-0.5t)^{-2} A' + (-2)(50-0.5t)^{-3} (-\frac{1}{2}) A = 10 (50-0.5t)^{-2} \]

\[ \Rightarrow A = 10 (50-0.5t)^2 \int (50-0.5t)^{-2} dt \]

** Finish **

(Recall IC A(0) = 0)

---


A 12-volt battery is connected in a series circuit in which the inductance \( \frac{1}{2} \) henry and the resistance 10 ohms. Determine the current \( i \) if the initial current is zero.
**Solution 1**

\[ R = 10 \text{ ohms} \]

\[ L = \frac{1}{2} \text{ H} \]

\[ E(t) = R \frac{dq}{dt} + L \frac{di}{dt} \quad (?) \]

Is this model correct?

**After Class**

**Answer:** Recall that \( \frac{dq}{dt} = i \), so eq \((?)\) becomes

\[ L \frac{di}{dt} + Ri = E(t) \quad \text{and in this example} \]

\[ \frac{1}{2} \frac{di}{dt} + 10i = 12 \]

or

\[ \frac{di}{dt} + 20i = 24 \quad (*) \]

**2.** This is the model in \textit{Std. Form}. Solve the model:

\[ a. \quad i(t) = e^{20t} \]

\[ b. \quad e^{20t} \frac{di}{dt} + 20 e^{20t} i = 24 e^{20t} \]

\[ (e^{20t} \cdot i)' = 24 e^{20t} \quad \text{Ck-mentally!} \]

\[ e^{20t} i = \frac{24}{20} e^{20t} + C \]

\[ \therefore \ i(t) = \frac{6}{5} + C e^{-20t} \]

**c.** The IC is \( i(0) = 0 : 0 = i(0) = \frac{6}{5} + C \), so \( C = -\frac{6}{5} \)

**d.** The model is

\[ i(t) = \frac{6}{5} - \frac{6}{5} e^{-20t} \]

This predicts the current for time \( t > 0 \).