I. Quiz in class. Finish
\[ A = 10 \int (50 - 0.5t)^2 (50 - 0.5t)^{-2} \, dt \]

\[ A(0) = 0. \]

Find \( A(t) \). → See next page for solution.

II. Direction Fields (59.1: p. 422).

A. If a ODE can be put into the form
\[ \frac{dy}{dx} = f(x, y), \]

then we can get an idea of what the graph looks like by using the direction field (slope field) method.

B. S.M. Example. \[ \frac{dy}{dx} = xy \]

\[ \text{Soln:} \begin{array}{ccc}
  x & y & xy \\
  0 & * & 0 \\
  * & 0 & 0 \\
  1 & 1 & 1 \\
  1 & 2 & 2 \\
  1 & 3 & 3 \\
  2 & 1 & 2 \\
  2 & 2 & 4 \\
  2 & 3 & 6 \\
  2 & 4 & 8 \\
  3 & 2 & 6 \\
\end{array} \]

IC \( y(0) = 1.5 \)

We looked at some Dir. Fields created in MAPLE and we solved our Mixture problem (began yesterday and finished today via the quiz above) using MAPLE. Then we looked at the Dir. Field for that problem.

* S.M. means "Simple Minded."
Outside class:

Solution to Quiz 5.21.09: Solve

\[ A = 10 \int (50 - 0.5t)^2 \left(50 - 0.5t\right)^{-2} dt \quad A(0) = 0 \]

\begin{align*}
\text{Solu} & \quad \int -1 \quad \text{d}u = 50 - 0.5t \quad \text{d}u = -0.5dt \quad \therefore -2du = dt \\
\text{and} & \quad \int (50 - 0.5t)^{-2} dt = -2 \int u^{-2} du = -2 \left(\frac{u^{-1}}{-1} + C_1\right) \\
= & \quad 2u^{-1} + C_2 \quad (C_2 = -2C_1) \\
= & \quad \frac{2}{50 - 0.5t} + C_2
\end{align*}

\[ A = 10 \left(50 - 0.5t\right)^2 \left\{ \frac{2}{50 - 0.5t} + C_2 \right\} \]

\[ A = 20 \left(50 - 0.5t\right) + C_3 \left(50 - 0.5t\right)^2 \quad (C_3 = 10C_2) \]

GenSol: \[ A(t) = 1000 - 10t + C \left(50 - 0.5t\right)^2 \]

or \[ C_3 \left(50 - 0.5t\right)^2 = C_4 \left(100 - t\right)^2 \]

\[ = \frac{C_3}{4} \left(100 - t\right)^2 = C_4 \left(100 - t\right)^2 = C \left(100 - t\right)^2 \]

\[ A(t) \text{ can be written as: } A(t) = 1000 - 10t + C \left(100 - t\right)^2 \]

And since \( (100 - t)^2 = (-100 + t)^2 \), my answer can be written

\[ A(t) = 1000 - 10t + C \left(-100 + t\right)^2 \]

Which is the same as the MAPLE solution.

Now to solve the IVP, the IC is \( A(0) = 0 \). Thus

\[ 0 = A(0) = 1000 + C \left(-100\right)^2 = 1000 + 10000C \]

Thus \[ C = \frac{-1000}{10000} = \frac{-1}{10} \]

So the solution is: \[ A(t) = 1000 - 10t - \frac{1}{10} \left(-100 + t\right)^2 \]