I. We discussed vector spaces & the basis vectors for vector spaces.

II. \( \text{Thm} \ 4.4 \): Criterion for Linearly Independent

\[ \text{Solutions, p. 122.} \]

\( \text{IF} \) \( y_1, y_2, \ldots, y_n \) are \( n \)-solutions of an \( n \)-th order HLODE on an interval \( I \)

\( \text{THEN} \) The set of solutions \( \{y_1, y_2, \ldots, y_n\} \) are L.I.

\( \iff \) \( W(y_1(x), y_2(x), \ldots, y_n(x)) \neq 0 \quad \forall \ x \in I. \)

\( \text{for all} \)

B. Compare \( \text{Thm} \ 4.4 \) with \( \text{Thm} \ 4.2 \) (p. 117).

C. \( \text{Def} \nolimits \ 4.3 \) (p. 123) A L.I. set of \( n \)-solutions to an \( n \)-th order HLODE on \( I \) is said to be a fundamental set of solutions.

D. \( \text{Thm} \ 4.5 \) (p. 123) If \( \{y_1, \ldots, y_n\} \) is a fund. set of solns. to an \( n \)-th order HLODE on \( I \) and if \( Y(x) \) is any soln. to the HLODE, then \( \exists \ C_1, C_2, \ldots, C_n \) such that \( Y(x) = \sum_{i=1}^{n} C_i y_i(x) \).
Example: $y_1 = \sin(2x)$ and $y_2 = \cos(2x)$ are solutions of

$$y'' + 4y = 0 \quad (\ast).$$

(You can verify this)

Now let's check the Wronskian.

$$W(\cos(2x), \sin(2x)) = \begin{vmatrix} \cos(2x) & \sin(2x) \\ -2\sin(2x) & 2\cos(2x) \end{vmatrix}$$

$$= \cos(2x) \cdot 2\cos(2x) - (-2)\sin(2x) \cdot \sin(2x)$$

$$= 2\cos^2(2x) + 2\sin^2(2x) = 2(\cos^2(2x) + \sin^2(2x))$$

$$= 2 \cdot 1 = 2 \neq 0 \quad \text{on} \quad (-\infty, \infty) \quad \text{so} \quad I = (-\infty, \infty).$$

$\therefore \{y_1, y_2\}$ are L.I.

$\therefore \{y_1, y_2\}$ is a fundamental set of solutions to \((\ast)\).

$\therefore y = c_1 \sin(2x) + c_2 \cos(2x)$ is the General Solution to \((\ast)\).