1. If I pick up a Calculus book, then I shall hurt my back.

2. If the Wronskian of 3 functions is not equal to zero (for at least one \( x \in I \)), then the 3 functions are linearly independent on \( I \).

3. Compare 1&2: what do they say vs. what don't they say?

II. In § 5.1: SHM, p. 184.

**A.** "MODEL" \( \frac{d^2x}{dt^2} + \omega^2 x = 0 \quad x(0) = \alpha \quad \frac{dx}{dt}(0) = \beta \)

\[ x'' + \omega^2 x = 0 \quad x(0) = \alpha, \quad x'(0) = \beta \]

\[ \ddot{x} + \omega^2 x = 0 \quad x(0) = \alpha, \quad \dot{x}(0) = \beta \]

**B.** Solve

1. AuxEq.: \( m^2 + \omega^2 = 0 \)

2. Solve AuxEq.: \( m^2 = -\omega^2 \quad m = \pm i \omega = 0 \pm \omega i \)

3. GenSol: \[ x = c_1 \cos(\omega t) + c_2 \sin(\omega t) \]

4. ICs

a. \( \dot{x} = -\alpha c_1 \sin(\omega t) + \omega c_2 \cos(\omega t) \)

b. \( \alpha = x(0) = C_1 \quad C_1 = \alpha \)

C. \( \beta = \dot{x}(0) = \omega c_2 \quad C_2 = \beta / \omega \)

\[ x = \alpha \cos(\omega t) + \frac{\beta}{\omega} \sin(\omega t) \]
Let's expand

Recall

\[ y = \sin (P \pm Q) \]
\[ = \sin P \cos Q \pm \cos P \sin Q \]
\[ y = \cos (P \pm Q) \]
\[ = \cos P \cos Q \mp \sin P \sin Q \]

\[ x(t) = A \sin (wt + \phi) \]
\[ = A \left[ \sin (wt) \cos (\phi) + \sin (\phi) \cos (wt) \right] \]
\[ = (A \sin \phi) \cos (wt) + (A \cos \phi) \sin (wt) \]

\[ x(t) = c_1 \cos (wt) + c_2 \sin (wt) \]

\[ \therefore c_1 = A \sin \phi \quad \text{and} \quad c_2 = A \cos \phi \quad (A). \]

\[ \sin \phi = \frac{c_1}{A}, \quad \cos \phi = \frac{c_2}{A} \]

From \((\star)\) \[ c_1^2 + c_2^2 = A^2 \sin^2 \phi + A^2 \cos^2 \phi = A^2 \]
\[ \therefore A = \sqrt{c_1^2 + c_2^2} \]

\[ \tan \phi = \frac{\sin \phi}{\cos \phi} = \frac{c_1/A}{c_2/A} = \frac{c_1}{c_2} \]
\[ \phi = \arctan \frac{c_1}{c_2} \]
Examples.

5.5.1: p. 189: #9. The period of free undamped oscillations (S#M) of a mass on a spring is \( \frac{\pi}{4} \) seconds. If the spring constant is \( 16 \frac{lb}{ft} \), what is the numerical value of the weight?

**Solution:**

1. \( P = \frac{\pi}{4} \). [Recall \( y = \cos(Bx) \)]
   \[ \text{Period} \quad P = \frac{2\pi}{B} \]
   \[ \frac{\pi}{4} = \frac{2\pi}{B} \]
   \[ B = 8 \]

2. \( \omega = \sqrt{\frac{k}{m}} \)

3. \( \omega = \sqrt{8} = \sqrt{16} \)

4. So \( 64 = \frac{16}{m} \)
   \[ m = \frac{16}{64} = \frac{1}{4} \]
   \[ w = mg \quad (g \approx 32) \]
   \[ w = \frac{1}{4} \times 32 = 8 \]

5. The weight is approximately 8 lb.