§ 1.1 Basic Def's (p.2)

A. ODE. (Ordinary Differential Equation) is an equation having one or more ordinary derivatives in it.

B. PDE. (Partial Diff. Eq.) is ....

C. Classification by
   1. Type (ODE/PDE)
   2. Order ("Highest Derivative")
   3. Linearity (vs Non-Linearity)

D. Examples of Linear/Nonlinear Eqns.
   1. \(3y' + 2y = \sin(x)\) Linear
      \(x \, dx + y \, dy = 0\) Linear
      \(\frac{dx}{dt} + 7x(t) = 2\)  

   2. \((\frac{dy}{dx})^2 + 3y = 0\) Non-linear

   3. \(\frac{d^2y}{dx^2} + 3y = 0\) Linear

E. A solution to a DE is a \textbf{FUNCTION}.
   It has a \textbf{domain of applicability}.
[1] Solve: \( \int x \, dx = \frac{1}{2}x^2 + C \) \( \circ \) infinite number of solutions

[2] Consider this: \[ y'' - 5y' + 6y = 0 \]
(\text{where } y = y(x) \text{ on } (-\infty, \infty))

[1] Suppose I say \( y_1 = e^{2x} \) is a solution.

Verification:
\[ y_1' = 2e^{2x}, \quad y_1'' = 4e^{2x} \]

\[ y_1'' - 5y_1' + 6y_1 = (4e^{2x}) - 5(2e^{2x}) + 6(e^{2x}) = 0 \checkmark \]

[2] Suppose I say \( y_2 = 20e^{2x} \) is a solution.

So even \( y = c_1 e^{2x} \) is a solution for \( c_1 \in \mathbb{R} \)

[1] Also \( y_3 = c_2 e^{3x} \) is a solution for \( c_2 \in \mathbb{R} \)

Verification:
\[ y_3' = 3c_2 e^{3x} \quad \text{and} \quad y_3'' = 9c_2 e^{3x} \]

\[ y_3'' - 5y_3' + 6y_3 = (9c_2 e^{3x}) - 5(3c_2 e^{3x}) + 6(c_2 e^{3x}) = c_2 [9 - 15 + 6] = 0 \checkmark \]

[1] : \{ c_1 e^{2x}, c_2 e^{3x} \} is a set of solutions to the ODE.
Also (as it turns out because of the linearity of the ODE), the function
\[ c_1 e^{2x} + c_2 e^{3x} \]
is also a solution to the ODE for any \( c_1, c_2 \in \mathbb{R} \).

\[ \epsilon \text{ epsilon} \]
\[ \in \text{ is an element of... such as } 5 \in \mathbb{N} \] "5 is an element of the set of natural numbers, "