I

A. Items from "Entrance Exam"

#2. This is how the answer STARTS. Don't forget it.

\[ F(x) = \text{etc. etc...} \]

#3. \[
\int \frac{dx}{\sqrt{1-x^2}} = \int \frac{\cos \theta}{\cos \theta} \, d\theta
\]

\[ = \text{etc. etc.} \]

\[ = \sin^{-1}(x) + C \]

Notice: Box Answer

Best Final Answer:

\[ \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x) + C \]

II

In §2.1 — p. 30.

A. Thm. 2.1: EXISTENCE of a UNIQUE SOL to an IVP. (Initial Value Problem)

Hypotheses: IF

1. \( R: [a, b] \times [c, d] \) "Cartesian Product"

\[
[a, b] \times [c, d] = \{ (x, y) | a \leq x \leq b \text{ and } c \leq y \leq d \}
\]

\[ y \]
\[ d \]
\[ c \]
\[ a \]
\[ b \]
\[ R \]
\[ \text{The "interior" of the rectangle} \]

is such that \( (x_0, y_0) \in \text{int } R \) and

2. \( z = f(x, y) \) is continuous on \( R \) and

3. \( \frac{df(x, y)}{dy} \) is "" "" \( R \)
THEN there exists an interval, I, "centered at $x_0"$

and a unique function $y(x)$ defined on I such that $y$ satisfies the IVP,

$$\frac{dy}{dx} = f(x, y) \quad \text{and} \quad y(x_0) = y_0.$$  

$I$ is on the $x$-axis, and the function $y$ is defined on $I$ (guaranteed by the Thm) so that

$$\frac{dy}{dx} = f(x, y)$$

AND $y(x_0) = y_0$.

III §2.2. Separable Variables.

We shall start here tomorrow. This TECHNIQUE — Separation of Variables is probably the most basic technique for solving ODEs WHEN IT IS APPLICABLE!

Read the "gray paragraph" on p. 31!
PREVIEW of the TECHNIQUE

1. \( \frac{dy}{dx} = x \) ← Here's the "problem."

\[ dy = x \, dx \] ← "separate" the variables

\[ \int dy = \int x \, dx \] ← This is the way it "looks"

\[ y = \frac{x^2}{2} + C \] ← Integrate both sides

Write anti-derivatives.

(Why is there only one "C"?)

2. \( \frac{dy}{dx} = x \)

\[ \int \frac{dy}{dx} \, dx = \int x \, dx \] ← Here's what "really happens."

\[ y = \frac{x^2}{2} + C \]

3. **IVP:** \( \frac{dy}{dx} = x \) ODE; \( y(1) = 3 \) IC

**Solve:**

1. \[ \int \frac{dy}{dx} \, dx = \int x \, dx \] ; \[ y = \frac{x^2}{2} + C \]

Initial Condition

2. IC: \( 3 = y(1) = \frac{1}{2} + C \) \[ \Rightarrow C = \frac{5}{2} \]

3. \[ y = \frac{x^2}{2} + \frac{5}{2} \]