II. From § 2.2 (Sep. Var.) p. 38. Solve via Sep. Var.

A. RE: #5

\[(x+1) \frac{dy}{dx} = x + 6\]

Solution


\[dy = \frac{x+6}{x+1} dx\]

2. \[x+1 \left| \frac{x+6}{x+1} \right| \quad \therefore \quad dy = \left(1 + \frac{5}{x+1}\right) dx\]

yada, yada, yada...

B. RE: #11.

\[\frac{dx}{dy} = \frac{x^2y^2}{1+x}\]

Solution

1. \[\frac{1+x}{x^2} \quad dx = y^2 dy\]

\[(x^2 + \frac{1}{x}) \quad dx = y^2 dy\]

yada, yada, yada...

III. § 2.3: Homogeneous Eqs of Deg. n. (p. 48).

A. An ODE of the form

\[M(x,y) \quad dx + N(x,y) \quad dy = 0\]  \hspace{1cm} (8)

is by def. HOMOGENEOUS of DEG. "n" iff

Both \(M\) and \(N\) are homogenous functions of the same degree, \(n\).

B. If \(Mdx + Ndy = 0\) is homogeneous of deg \(n\), then...
Try one of these substitutions
\[ y = ux \quad \text{or} \quad x = vy, \quad u, v \text{ exclusive} \]

This will yield a separable ODE.

**Examples.**

1. § 2.3: p. 45: #11 Solve via subs.

\[ (x-y)dx + x \, dy = 0 \quad (\star) \]

Sols:

a. Let \( y = ux \quad \Rightarrow \quad dy = u \, dx + x \, du \)

b. Subs. into (\star) \quad \therefore \quad u = \frac{y}{x}

\[ (x-ux) \, dx + x (u \, dx + x \, du) = 0 \]
\[ x (1-u) \, dx + x (u \, dx + x \, du) = 0 \quad \text{if} \quad x \neq 0 \]
\[ (1-u) \, dx + (u \, dx + x \, du) = 0 \]

\[ dx + x \, du = 0 \quad (\star \star) \]

C Solve the separable (\star \star)

\[ \therefore \quad \frac{dx}{x} = -du \]
\[ \therefore \quad \int \frac{dx}{x} = - \int du \Rightarrow \quad \ln |x| = -u + C \]

\[ \ln |x| = -\frac{y}{x} + C \quad \text{OK} \]
\[ \therefore \quad y = -x \ln |x| + Cx \quad \text{"better"} \]
§ 2.4: EXACT EQUATIONS (p. 46).

A. We are dealing again with a "1-form"

\[ M \, dx + N \, dy = 0 \quad (*) \]

B. We say \((*)\) is EXACT iff

\[ \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \]

\[ \text{CRITERION for EXACTNESS!} \]

\[ \text{Note: In IIIC above --} \]

Eqn. \((*)\):

\[ (x-y) \, dx + x \, dy = 0 \]

is, in fact, homogeneous of degree 1, since \(M(x,y) = x-y\)

and \(M(tx, ty) = (tx) - (ty) = t(x-y) = tM(x,y)\)

\[ \therefore M \text{ is homogeneous of degree } 1. \]

AND \(N(x,y) = x\) and \(N(tx, ty) = (tx) = tx = tN(sy)\)

\[ \therefore N \text{ is homogeneous of degree } 1. \]

Since both \(M \neq N\) are homogeneous of the same degree (i.e., 1), Eqn \((*)\) is homogeneous of degree 1.