CH 4 LINEAR ODEs OF HIGHER ORDER (p. 111)

Start § 4.1 - PRELIM. THEORY.

§ 4.1.1: Initial Value & Boundary Value Problems (p. 112).

Examples of Eqns.

a) \( y' + y = \frac{1}{x} \)
   Linear Non-Homog. ODE of deg 1

\( (y')^2 + y = \frac{1}{x} \)
   Non-Linear

\( y' + \sin(y) \cdot y = \frac{1}{x} \)  
  "

\( y' + (\cos x) \cdot y^2 = 0 \)  
  "

b) \( 5 \frac{d^2 s}{dt^2} - \sqrt{t} \frac{ds}{dt} + 0.1 s = \frac{1}{2} \sin(t) \)
   Non-Homog. LODE of deg 2.

c) \( y''' - \cosh(x) y' - 3y = 5 \)
   Non-Homog. LODE of deg 3

Example (p. 113). Show "c" a constant.

\( \phi(x) = cx^2 + x + 3 \) is a soln to

IVP: \( x^2 y''' - 2xy' + 2y = 6 \) (not given) \( y(0) = 3, y'(0) = \) (cont...?)
Solve

$\phi \prime \prime (x) = 2cz + 1$  

$\phi \prime \prime \prime (x) = 2c$

2 Subs into LHS of (**)

$x^2 \phi \prime \prime - 2x \phi \prime + 2\phi = x^2 (2c) - 2x (2cz + 1) + 2(2c^2 + x^3)$

$= 2cx^2 - 4cxz - 2x + 2c^2x^2 + 2x + 6$

$= 6 \quad \checkmark$  

(3) $\phi$ is a soln of the 2nd order LODE

and $\phi(0) = 0 \cdot 0 + 0 + 3 = 3 \quad \checkmark$

and $\phi'(0) = 2c(0) + 1 = 1 \quad \checkmark$

and $\phi''(0) = 2c(0) + 1 = 1 \quad \checkmark$

C In General A 4th order LODE looks like this.

$\frac{d^4 y}{dx^4} + a_3(x) \frac{d^3 y}{dx^3} + a_2(x) \frac{d^2 y}{dx^2} + a_1(x) \frac{dy}{dx}$

$+ a_0(x)y = f(x)$

And an $n$-th Order LODE:

$\frac{d^n y}{dx^n} + a_{n-1}(x) \frac{d^{n-1} y}{dx^{n-1}} + \ldots + a_2(x) \frac{d^2 y}{dx^2} + a_1(x) \frac{dy}{dx} + a_0(x)y = f(x)$

D Thm 4.1: p. 112: Given an $n$-th order LODE such that on some interval of interest $(a, b)$ $a_n, a_{n-1}, \ldots, a_0$ and $f$ are all continuous i.e. $a_n(x)$ is NEVER ZERO on the interval $(a, b)$ and if $x_0 \in (a, b)$,

Then there exists a unique soln to the IVP on $(a, b)$.  

interval
Boundary Value Problem. (p. 114)

2nd order

\[ a_2(x) \frac{d^2 y}{dx^2} + a_1(x) \frac{dy}{dx} + a_0(x) y = g(x) \]

and \( y(a) = y_0 \) and \( y(b) = y_1 \) \( \in \) bdry values

\[ xy'' - y' = 0 \] \( \neq \) bdry cond. \( y(0) = 1, \; y'(1) = 0 \)

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class ends

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Soll: Let's look for an IF for (*) since \( y'' \) is the der.

of \( y' \)

\[ y'' - \frac{1}{x} y' = 0 \quad P(x) = -\frac{1}{x}, \; I(x) = -\ln|x| \quad \text{take } x > 0 \]

\[ \mu(x) = e^{-\ln x} = e^\ln \frac{1}{x} = \frac{1}{x} \]

\[ \frac{1}{x} y'' - \frac{1}{x^2} y' = 0 \]

\[ (\frac{1}{x} y')' = 0 \]

\[ \frac{1}{x} y' = C \]

\[ y' = Cx \]

\[ y = C_1 x^2 + C_2 \] \( (***) \) \[ C_1 = \frac{C}{2} \]

Now we have discovered a solution (we still need to check) for(*) on the interval \( x > 0 \), but an interesting observation is that this function \( (***) \) seems to be a solution for all real numbers \((-\infty, \infty)\). Let's check.

(cont...)
If \( y = c_1 x^2 + c_2 \), then \( y' = 2c_1 x \) and \( y'' = 2c_1 \).

Subs into LHS of (4): \( xy'' - y' = x(2c_1) - (2c_1 x) = 0 \); RHS(4) ✓

Thus \( y \) (or perhaps I should call it \( y_1 \) or \( \phi \)) is a sol'n to (4).

3. Now check the bdry values (boundary values) to determine \( c_1, c_2 \).

   \[ 1 = y(0) = c_1(0)^2 + c_2 \implies c_2 = 1 \]

   \[ \therefore y = c_1 x^2 + 1 \]

And

   \[ 6 = y'(1) = 2c_1(1) = 2c_1 \implies c_1 = 3 \]

\[ \therefore y = 3x^2 + 1 \]

4. Summary: \( y = 3x^2 + 1 \) is a sol'n to the given bdry value problem.

However, we have no guarantees that this sol'n is unique. ❍