We reviewed additions made to yesterday's notes.

From § 5.2.

**Complementary Events.**

**Example.** Suppose

\[ S = \{ a, b, c, 7, 1, 4, *, c \} \]  \( n(S) = 8 \)

Let \( E_1 = \{ a, b, c \} \)

"Event #1"

The **complement** of event \( E_1 \), denoted by \( E_1^c \), is all elements (members) of \( S \) which are not in \( E_1 \).

\[ \therefore E_1^c = \{ 7, 1, 4, *, c \} \]

\[ E_2 = \{ b, 4 \} \implies E_2^c = \{ a, c, 7, 1, *, c \} \]

**Complementary Probabilities**

In Example above.

\[
P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{8}
\]

\[
P(E_1^c) = \frac{n(E_1^c)}{n(S)} = \frac{5}{8}
\]

Important: Notice that \( \frac{3}{8} + \frac{5}{8} = \frac{8}{8} = 1 \)

[Continued]
My point here is that if I know what $P(E_i)$ is, I don't have to list $E_i^c$ to figure its probability.

\[
P(E_i^c) = 1 - P(E_i) \quad (*) \text{ACTIVE VOCAB.}
\]

\[
(P(E) + P(E^c) = 1) \quad \text{idea here}
\]

\[E_1 \cap E_2 = \emptyset \quad \text{ACTIVE VOCAB.} \quad (**)
\]

**CLASS ENDS...**

**C** Note: The formula for the probability of an event $E$

\[
P(E) = \frac{n(E)}{n(S)} \quad \text{is ACTIVE VOCAB.} \quad (!) \quad (***)
\]

**D** Formula $(*)$, above, is particularly useful in "solving" the probability of the form "... at least one...."

Right now, all I can give you is a really simple example, which could easily be solved with a simple, straightforward method. But here, we illustrate a very important
technique:

First, note that $P(E^c) = 1 - P(E)$ (from prev pg) can also be written

$$P(E) = 1 - P(E^c) \quad (*)$$

Second, note that the complement of "...at least one...." is "none...."

Thus, in the "Three question quiz" example — I remind you

$$S = \{ CCC, CCI, CIC, ICC, CII, ICI, IIC, III \}$$

and $n(S) = 8$  \[ B \]

If the question is, "What is the probability that the student gets at least one question right?"

then the event is "...gets at least one question right."

This is $E$.

What is $E^c$?  $E^c$ is "the student gets no question right."

Another way of saying the same thing is:

"the student gets all questions wrong."

This is $E^c$, and it is easy (I hope) to see that

$$P(E^c) = \frac{n(E^c)}{n(S)} = \frac{1}{8}.$$ 

\[ \therefore \] $P(E) = P(\text{at least one right}) = 1 - P(E^c) = 1 - \frac{1}{8} = \frac{7}{8}$.