II Probability

- Theoretical
- Empirical

Sample Space

Just think about it

Events

\[ P(E) = \frac{n(E)}{n(S)} \]

Active Vocab.

II Disjoint Events

\[ E_1 \cap E_2 = \emptyset \text{ (no "overlap")} \]

III Compound Event

\[ P(A \text{ or } B) = P(A \cup B) \]

Suppose \( A = \{1, 2, 3\} \) \( B = \{2, 3, 4, 5\} \)
\( S = \{1, 2, 3, 4, 5, 6, 7, 8\} \)

\[ P(A \text{ or } B) = \frac{n(A \cup B)}{n(S)} = \frac{5}{8} \]

\[ P(A \text{ and } B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{8} = \frac{1}{4} \]
\[ P(A) = \frac{n(A)}{n(s)} = \frac{3}{8} \]
\[ P(B) = \frac{n(B)}{n(s)} = \frac{4}{8} = \frac{1}{2} \]
\[ P(A) \times P(B) = \frac{3}{8} \times \frac{1}{2} = \frac{3}{16} \]

To repeat \( P(A \text{ and } B) = \frac{1}{4} \)

Here \( P(A \text{ and } B) \neq P(A) \times P(B) \)

In such a situation we say that the events \( A \) and \( B \) are **not disjoint** \( \text{ and not independent} \). **They are dependent.**

**In general a formula that always works** is
\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

Why? \( P(A) = \frac{3}{8}, P(B) = \frac{1}{2} \) to repeat.

\[ P(A \text{ or } B) = \frac{n(A \text{ or } B)}{n(s)} = \frac{5}{8} \]

\[ P(A) + P(B) = \frac{3}{8} + \frac{1}{2} = \frac{3}{8} + \frac{4}{8} = \frac{7}{8} \]

**The formula that always works for "or" problems is**
\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]
So if A and B are DISJOINT
Then $A \cap B = \emptyset$
and $n(A \cap B) = n(\emptyset) = 0$
\[ \therefore P(A \text{ and } B) = \frac{n(\emptyset)}{n(S)} = 0 \]
\[ \therefore \text{ If } A \text{ and } B \text{ are DISJOINT}, \]
\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]
\[ = P(A) + P(B) - 0 \]
\[ P(A \text{ or } B) = P(A) + P(B) \text{ if DISJOINT} \]

After Class

Note in the example above
\[ P(A \text{ or } B) = \frac{5}{8}, \quad P(A) + P(B) = \frac{7}{8} \]
and $P(A \text{ and } B) = \frac{1}{4}$

So let's see if the "OR"-formula works for this example.
\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]
\[ \frac{5}{8} \overset{?}{=} \frac{3}{8} + \frac{1}{2} - \frac{1}{4} \]
\[ \frac{5}{8} \overset{?}{=} \frac{3}{8} + \frac{4}{8} - \frac{2}{8} \]
\[ \frac{5}{8} \overset{?}{=} \frac{3 + 4 - 2}{8} \]
\[ \frac{5}{8} = \frac{5}{8} \rightarrow \text{YES!} \]