The function \( P(t) = P_0 e^{kt} \), \( P_0 \) is the population at time \( t = 0 \), \( P \) is the population after time \( t \), \( k \) is the exponential growth rate

The doubling time, \( T \) of a population is:

\[ T = \frac{\ln 2}{k}. \]

See Example 1 page 409.

1. In 2004, the world population was 6.4 billion. The exponential growth rate was 1.1% per year and can be modeled with the exponential growth function:

\[ P(t) = 6.4 e^{0.011t} \] where \( t \) is the number of years since 2004 and \( P(t) \) is in billions.

   a) Estimate the population of the world in 2006 and in 2016.
   
   b) When will the world population be 8 billion?
   
   c) Find the doubling time.

2. The current natural population growth rate of the United States is 1.9% per year. The population in November of 2005 was about 295.7 million. This growth can be modeled with the function:

\[ P(t) = 295.7 e^{0.019t} \] where \( t \) is the number of years since 2005 and \( P(t) \) is in millions.

   a) Estimate the population of the United States in 2006 and in 2011.
   
   b) When will the United States population be 350 million?
   
   c) Find the doubling time.

The function \( P(t) = P_0 e^{kt} \), \( P_0 \) is the amount invested at time \( t = 0 \), \( P \) is the amount after \( t \) years, \( k \) is the interest rate compounded continuously

The doubling time, \( T \) of the amount is:

\[ T = \frac{\ln 2}{k}. \]

See Example 2 pages 410-412.

3. Suppose that $10,000 is invested at an interest rate of 5.4% compounded continuously.

\[ P(t) = 10,000 e^{0.054t} \]

   a) What is the balance after 1 Year? 5 Years? 10 Years?
   
   b) When will the account have $15,000?
   
   c) Find the doubling time.

(Revised 8/08)
4. Suppose that $60,000 is invested at an interest rate of 3.75% compounded continuously.

\[ P(t) = 60,000 e^{0.0375t} \]

a) What is the balance after 1 Year? 5 Years? 10 Years?

b) When will the account have $100,000?

c) Find the doubling time.

The function \( P(t) = P_0 e^{-kt}, \ k > 0 \) is a model of the decay of a radioactive substance.

\( P_0 \) is the amount of a substance at time \( t = 0 \)

\( P \) is the amount still radioactive after time \( t \)

\( k \) is a positive constant that is the decay rate.

The half-life, \( T \), of a substance is the time that it takes for \( \frac{1}{2} \) of the substance to cease to be radioactive.

\[ T = \frac{\ln \left( \frac{1}{2} \right)}{-k} = -\ln \left( \frac{1}{2} \right) = \frac{\ln \left( \frac{1}{2} \right)}{-k} = \frac{\ln(2)}{k} \]

See Example 5 page 415.

5. Polonium has a Half-life \( T \) of 3 minutes.

\( P(t) = P_0 e^{-k3} \)

a) Find \( k \), the decay rate.

b) Write the equation \( P(t) \) with the constant \( k \).

6. Lead has a Half-life \( T \) of 22 years.

\( P(t) = P_0 e^{-k22} \)

a) Find \( k \), the decay rate.

b) Write the equation \( P(t) \) with the constant \( k \).

7. Iodine-131 has a decay rate \( k \) of 9.6% per day.

\( P(t) = P_0 e^{-0.096t} \)

a) Find \( T \), the Half-life of Iodine-131.

(Revised 8/08)
1. a) In 2006, \( t = 2 \) \[ P(t) = 6.4 e^{0.011(2)} = 6.54 \text{ Billion} \]
   In 2016, \( t = 12 \) \[ P(t) = 6.4 e^{0.011(12)} = 7.3 \text{ Billion} \]
   b) in 20.29 years Or about the middle of 2024
   c) \[ T = \frac{\ln 2}{0.011} = 63.01 \text{ years.} \]

2. a) In 2006, \( t = 1 \) \[ P(t) = 295.7 e^{0.019(1)} = 301.37 \text{ Million} \]
   In 2011, \( t = 6 \) \[ P(t) = 295.7 e^{0.019(6)} = 331.41 \text{ Million} \]
   b) In 8.87 years Or towards the end of 2013
   c) \[ T = \frac{\ln 2}{0.019} = 36.48 \text{ years.} \]

3. a) 1 Year \( t = 1 \) \[ P(t) = 10,000 e^{0.054(1)} = \$10,554.85 \]
   5 Years \( t = 5 \) \[ P(t) = 10,000 e^{0.054(5)} = \$13,099.65 \]
   10 Years \( t = 10 \) \[ P(t) = 10,000 e^{0.054(10)} = \$17,160.07 \]
   b) In 7.5 years
   c) \[ T = \frac{\ln 2}{0.054} = 12.84 \text{ years.} \]

4. a) 1 Year \( t = 1 \) \[ P(t) = 60,000 e^{0.0375(1)} = \$62,292.72 \]
   5 Years \( t = 5 \) \[ P(t) = 60,000 e^{0.0375(5)} = \$72,373.81 \]
   10 Years \( t = 10 \) \[ P(t) = 60,000 e^{0.0375(10)} = \$87,299.48 \]
   b) In 13.62 years
   c) \[ T = \frac{\ln 2}{0.0375} = 18.48 \text{ years.} \]

5. a) \( k = 0.2310 \) 23.1\% per minute
   b) \[ P(t) = P_0 e^{-0.2310 t} \]

6. a) \( k = 0.0315 \) 3.15\% per year
   b) \[ P(t) = P_0 e^{-0.0315 t} \]

7. a) \( T = 7.22 \text{ Days} \)

(Revised 8/08)