Sec. 61  Angles and Their Measure

Definition: Radian:

- \( \theta \) is \( 1 \) radian

- \( r = \frac{\text{radians}}{r} \)

\[ \theta = 2\pi k \quad \text{for an integer} \]

I. Convert between Degrees (°), Minutes (′), Seconds (″), and Decimal Forms

\[ 61°42'21'' \rightarrow \text{Degrees?} \]

\[ 61 + 42\left(\frac{1}{60}\right) + 21\left(\frac{1}{3600}\right) = 61.71° \]

\[ 29.411° \rightarrow \text{DMS?} \]

\[ 29° + 0.411(60') = 29.66 \]

\[ 29°24' + 0.66(60'') = 39.6 \approx 40 \]

\[ 29°24'40'' \]

\[ 44.01 \rightarrow \text{DMS?} \]

\[ 44° .01(60') = .6 = 0.6 \]

\[ 44°0' .6(60'') = 36 \]

\[ 44°0'36'' \]
II. Find the Arc Length of a Circle.

\[ \ell = r\theta \]  \( \theta \) must be in radians

1. \( \theta = \frac{1}{4} \text{ radian}, \ell = 6 \text{ cm}, \ r = ? \)

\[ \ell = r\theta \]

\[ 6 = \frac{1}{4}r \quad \text{multiply 4 to both sides} \]

\[ 24 = r \text{ cm} \]

III. Convert from Degrees to Radians and Radians to Degrees

1 revolution = \( 2\pi \) radians

Since, \( 180^\circ = \pi \),

Degrees to Rads

Multiply by \( \frac{\pi}{180^\circ} \)

\[ \text{Ex} \quad \frac{225^\circ}{1} \rightarrow \text{Rads?} \]

\[ \frac{45}{225^\circ} \left( \frac{\pi}{180^\circ} \right) = -\frac{5}{1} \left( \frac{\pi}{4} \right) = -\frac{5\pi}{4} \]

Rads to Degrees

Multiply by \( \frac{180^\circ}{\pi} \)

\[ \text{Ex} \quad \frac{5\pi}{12} \rightarrow \text{Degrees?} \]

\[ \frac{5\pi}{12} \left( \frac{180^\circ}{\pi} \right) = \frac{5}{1} \left( \frac{15^\circ}{1} \right) = 75^\circ \]

IV. Find the Area of a Sector of a Circle

\[ A = \frac{1}{2} r^2 \theta \]
\[ r = 3 \text{ meters}, \theta = 120^\circ, \quad A = ? \]

\[ A = \frac{1}{2} r^2 \theta \]

\[ \theta \text{ must be in radians} \quad 120^\circ \left( \frac{\pi}{180^\circ} \right) = \frac{2\pi}{3} \]

\[ A = \frac{1}{2} (3)^2 \left( \frac{2\pi}{3} \right) \]

\[ A = \frac{9\pi}{2} \left( \frac{2\pi}{3} \right) = \frac{3\pi}{2} m^2 \approx 9.425 m^2 \]

\[ \text{Distance between Cities} \]

City 1 is due North of City 2. Find the distance between City 1 (38°21' North latitude) and City 2 (30°20' North latitude).

Assume radius of Earth is 3960 miles.

\[ \theta = 38^\circ 21' - 30^\circ 20' = 8^\circ 1' = \left( 8 + \frac{1}{60} \right)^\circ \left( \frac{\pi}{180} \right) = 0.139917 \text{ radians} \]

\[ d = r \theta \]

\[ d = 3960 \cdot 0.139917 = 554.07 \text{ miles} \]

VI Find Angular (\( \omega \)) and Linear speed (\( v \)) of an Object traveling in Circular motion

\[ \omega = \frac{\theta}{t} \quad \text{hence } \omega \text{ is radians/second} \]

\[ v = \frac{d}{t} \quad v = \frac{\theta}{t} \]

\[ v = rw \]
A neighborhood carnival has a Ferris wheel whose radius is 30 ft. Time it takes for one revolution is 70 seconds. What is linear speed (in feet per second) of this Ferris wheel? What is the angular speed in radians per second?

Angular speed \( \omega \): 1 rev in 70 sec.

\[ \omega = \frac{\text{rev}}{\text{sec}} \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = \frac{2\pi}{35} \text{ rad/sec} \approx 0.09 \text{ rad/sec} \]

Linear speed \( v \):

\[ v = \omega r = \frac{2\pi}{35} \cdot 30 = \frac{6\pi}{7} \text{ ft/sec} = 2.69 \text{ ft/sec} \]

Ex A spin balancer rotates the wheel of a car at 480 revolutions per minute. If the diameter of the wheel is 26 inches, what road speed is being tested? (miles per hour).

Road speed \( v \) → linear velocity in miles per hour.

\[ r \text{ in miles; } \omega \text{ in rad/hr.} \]

\[ v = \omega r = \frac{480 \text{ rev}}{1 \text{ min}} \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{60 \text{ min}}{1 \text{ hr}} \right) = 57600\pi \text{ rad/hr.} \]

\[ r = \frac{13}{12} \text{ in.} \left( \frac{1 \text{ ft}}{12 \text{ in.}} \right) \left( \frac{5280 \text{ ft}}{1 \text{ hr}} \right) = \frac{13}{63360} \text{ miles} \]

\[ v = \frac{13}{63360} \cdot 57600\pi \approx 37.128 \text{ miles/hr.} \]