6.3 Trigonometric Functions of General Angles

I. Find the exact values of the trig fns for general $x$'s.

- **Example:**
  - Use unit circle for $x$'s that terminate on an axis like $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, etc.
  - Same for degree measures.
  - $\cos \frac{3\pi}{2} = 0 \rightarrow \sec \frac{3\pi}{2} = \frac{1}{0} = \text{undefined}$
  - $\sin \frac{3\pi}{2} = -1 \rightarrow \csc \frac{3\pi}{2} = \frac{1}{-1} = -1$
  - $\tan \frac{3\pi}{2} = \frac{\sin \frac{3\pi}{2}}{\cos \frac{3\pi}{2}} = \frac{-1}{0} = \text{undefined}$
  - $\cot \frac{3\pi}{2} = \frac{0}{-1} = 0$

1. $\cos (-2\pi)$
   - $\cos (-2\pi) = 1$

2. $\sec \left(-\frac{5\pi}{2}\right) = \frac{1}{\sin \left(-\frac{5\pi}{2}\right)}$

3. $\sin 90^\circ$
   - $\sin 90^\circ = 1$

I. B. Using reference angles to find the exact value.

**Hint:** Always use the x-axis as a side. Reference $x$ is adjacent to the x-axis and always positive.
use $45^\circ \Delta$ and $30^\circ, 60^\circ \Delta$ for any $\theta$ whose reference is either $45^\circ, 30^\circ,$ or $60^\circ$.

$45^\circ = \frac{\pi}{4}, \ 30^\circ = \frac{\pi}{6}, \ 60^\circ = \frac{\pi}{3}$

All trig functions positive in quadrant I.
Cosine positive in quadrant IV. Hence same for secant.
Sine " " " " II. " " " " cosecant
Tangent " " " " III. " " " " cotangent

*Use reference $\theta$ to find the exact value of each expression.

6. $\cos 210^\circ$

Reference $\theta = 210 - 180 = 30$

$\theta = 210^\circ - 180^\circ = 30$

Use reference $\theta$ $(30^\circ)$ and $\frac{\sin \theta}{\cos \theta}$ to find exact values.

$\sin 210^\circ = -\frac{1}{2}$
$\cos 210^\circ = -\frac{\sqrt{3}}{2}$
$\tan 210^\circ = \frac{1}{\sqrt{3}}$
$\csc 210^\circ = -2$
$\sec 210^\circ = -\frac{2}{\sqrt{3}}$
$\cot 210^\circ = \sqrt{3}$

Only tangent and cotangent positive in quadrant III.
\( 30^\circ = \frac{\pi}{6} \)

\( 60^\circ = \frac{\pi}{3} \)

\( 45^\circ = \frac{\pi}{4} \)

7. \( \cos \frac{2\pi}{3} \)

\( \frac{2\pi}{3} \) is in II

and reference \( \theta \) is \( \frac{\pi}{3} \) or \( 60^\circ \)

Cosine is \((-\)) in II

\[
\cos \frac{2\pi}{3} = -\frac{1}{2}
\]

8. \( \sin(-210^\circ) \)

\( -210^\circ \) in III

\( \theta \) - reference \( \theta \) is \( 210 - 180 = 30^\circ \)

\( \sin \) is \((+)\) in II

\[
\sin(-210^\circ) = -\frac{1}{2}
\]

9. \( \sec \left(-\frac{9\pi}{4}\right) \)

\( -\frac{9\pi}{4} \) in IV

Reference \( \theta \) is \( \frac{\pi}{4} \) or \( 45^\circ \)

Secant is \((+)\) in IV

\[
\sec \left(-\frac{9\pi}{4}\right) = \sqrt{2}
\]

Find the exact value of each of the remaining trig fns of \( \theta \)

9. \( \sin \theta = -\frac{5}{13} \), \( \theta \) in III
negative because sine is so in III

\[ 5^2 + x^2 = 13^2 \]
\[ 25 + x^2 = 169 \]
\[ x^2 = 144 \]
\[ x = 12 \]

\[ \sin \theta = -\frac{5}{13} \]
\[ \csc \theta = -\frac{13}{5} \]
\[ \cos \theta = -\frac{12}{13} \]
\[ \sec \theta = -\frac{13}{12} \]
\[ \tan \theta = \frac{5}{12} \]
\[ \cot \theta = \frac{12}{5} \]

(10) \[ \cos \theta = \frac{4}{5}, \quad 270^\circ < \theta < 360^\circ \]

\[ 4^2 + y^2 = 5^2 \]
\[ 16 + y^2 = 25 \]
\[ y^2 = 9 \]
\[ y = 3 \]

\[ \sin \theta = -\frac{3}{5} \]
\[ \csc \theta = -\frac{5}{3} \]
\[ \cos \theta = \frac{4}{5} \]
\[ \sec \theta = \frac{5}{4} \]
\[ \tan \theta = -\frac{3}{4} \]
\[ \cot \theta = -\frac{4}{3} \]

(11) \[ \cot \theta = -2, \quad \sec \theta > 0 \]

the only quadrant that cotangent is negative and secant is positive is IV. \[ \frac{5}{13} \]

\[ 2^2 + 1^2 = c^2 \]
\[ 4 + 1 = c^2 \]
\[ 5 = c^2 \]
\[ c = \sqrt{5} \]

\[ \sin \theta = -\frac{1}{\sqrt{5}} \]
\[ \csc \theta = -\sqrt{5} \]
\[ \cos \theta = \frac{2}{\sqrt{5}} \]
\[ \sec \theta = \frac{\sqrt{5}}{2} \]
\[ \tan \theta = -\frac{1}{2} \]
\[ \cot \theta = -2 \]