4.5 Applied Optimization Problems (Read book for more examples)

Optimization refers to local max or min.

1. Show that among all rectangles with an 8-m perimeter, the one with largest area is a square.

\[ 2l + 2w = 8 \quad \Rightarrow \quad A = lw \]

\[ l = \frac{8 - 2w}{2} \quad \Rightarrow \quad A = (4 - w)w \]

\[ l = 4 - w \quad \Rightarrow \quad A = 4w - w^2 \]

\[ A' = 4 - 2w \]

4 - 2w = 0

2w = 4

w = 2 \leq \text{critical value.} \]

\[ A' = 4 - 2w \quad \frac{+}{-} \quad 0 \quad 2 \quad 3 \]

Area maximizes when w = 2

If w = 2, \( l \) has to be 2m since \( l = 4 - w \).

Thus \( \frac{2}{2} \) is a square. \( \sqrt{\} \)

A rectangle has its base on the x-axis and its upper two vertices on the parabola \( y = 12 - x^2 \). What is the largest area the rectangle can have, and what are its dimensions?

**Draw a picture:**

```
\[y = 12 - x^2\]
```

**Length is** \( 12 - x^2 \)

**Width is** \( 2x \)

\[
A = 2x(12 - x^2)
\]

\[
A = 24x - 2x^3
\]

\[
A' = 24 - 6x^2
\]

\[
24 - 6x^2 = 0
\]

\[
6x^2 = 24
\]

\[
x^2 = 4
\]

\[
x = \pm 2
\]

\[
A' = 12 - 3x^2
\]

\[
\begin{array}{c}
-3 \\
-2 \hspace{1cm} 0 \hspace{1cm} 2 \hspace{1cm} 3
\end{array}
\]

**Area maximizes at** \( x = 2 \)

If \( x = 2 \), width is 4 and length is 8 and largest area is 32.
What are the dimensions of the lightest open-top right circular cylindrical can that will hold a volume of 1000 cm³?

Lightest means least amount of material.

Surface Area of can

\[ A = 2\pi r h + \pi r^2 \]

side of can only

\[ V = \pi r^2 h \]

\[ 1000 = \pi r^2 h \]

\[ h = \frac{1000}{\pi r^2} \]

\[ A = 2\pi r \left(\frac{1000}{\pi r^2}\right) + \pi r^2 \]

\[ A = \frac{2000}{r} + \pi r^2 \]

\[ A' = -\frac{2000}{r^2} + 2\pi r \]

\[ 0 = -2000r^2 + 2\pi r \]

\[ 0 = r^2(-2000 + 2\pi r^3) \]

\[ -2000 + 2\pi r^3 = 0 \]

\[ 2\pi r^3 = 2000 \]

\[ r^3 = \frac{2000}{2\pi} \]

\[ r = \sqrt[3]{\frac{1000}{\pi}} \approx 6.83 \text{ cm} \]

Surface Area minimizes at \( r = \sqrt[3]{\frac{1000}{\pi}} \)

If \( r = 6.83 \text{ cm}, \ h = 6.82 \text{ cm} \).